Leveraging Lotteries for School Value-Added: Bias Reduction vs. Efficiency

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Value-added Models

- Value-added models (VAMs): Used to estimate causal effects of teachers and schools on student achievement
- Typical VAM: OLS regression of test scores on school indicators and controls; relies on selection-on-observables assumption
- VAMs are central to policy decisions
  - Awards for good performers (TN, PA)
  - Punitive measures (NYC, New Orleans)
  - School report cards
  - NCLB waivers
- VAM assumptions are controversial
  - Teacher VAM debate (Rothstein 2010; Kane et al., 2013; Chetty et al., 2014; Rothstein 2014)
  - School VAMs have received less attention, despite increasing policy role (Deming 2014)

School Quasi-experiments

- Parallel strand of literature: Quasi-experimental evaluations of groups of schools
- Many districts use centralized assignment mechanisms based on the theory of market design (Boston, NYC, New Orleans, Denver)
- These mechanisms involve random tie-breaking within priority groups
- Other schools admit using independent lotteries or test score cutoffs
- Several studies have used admissions records to estimate causal effects:
  - Open enrollment lotteries (Cullen et al. 2006)
  - Charter schools (Abdulkadiroglu et al., 2011; Angrist et al., 2012, 2013a, 2013b; Dobbie and Fryer, 2013)
  - Magnet schools (Deming et al., 2014)
  - Exam schools (Abdulkadiroglu et al., 2014a)
  - Small high schools (Abdulkadiroglu et al., 2014b)
- We use quasi-experiments to validate/improve observational measures of school value-added

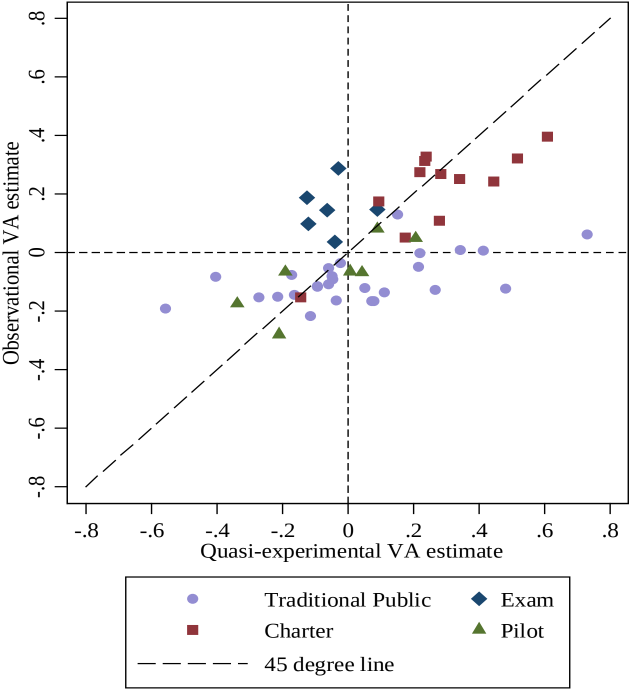
Our Approach

- We use data from Boston to estimate and compare quasi-experimental and observational value-added models
- Three goals:
  1. Develop methods for quasi-experimental VAM estimation
  2. Characterize extent of bias in observational VAMs
  3. Develop a combined measure of value-added that improves upon either observational or quasi-experimental estimates alone
- Observational estimates are precise but possibly biased; lottery-based estimates are unbiased but imprecise
- We develop a minimum mean squared error (MMSE) estimator that combines the advantages of each approach
- Methods may be useful in other settings involving tradeoffs between bias and precision

Preview of Findings

- Substantial bias in observational value-added estimates, both within and between school sectors
  - Available controls insufficient to eliminate differences in unobserved ability, e.g. between exam and traditional public students
  - Within-sector std. dev. of bias in math estimates is 0.1σ, large compared to variation in true value-added (0.16σ)
- MMSE estimator reduces error in VAM-based policies
  - 50% reduction in RMSE relative to traditional VAM
  - Misclassification rate for failing (lowest-quintile) schools falls from 49% to 27%
- Results establish the value of lottery-based and hybrid VAM estimation strategies
- We conclude with a summary of relationships between value-added, bias, and school oversubscription

Figure 3a: Observational and quasi-experimental math value-added estimates, by sector



Related Literature

- School lotteries (Abdulkadiroglu et al., 2011, 2014a, 2014b; Angrist et al., 2012, 2013a, 2013b; Cullen et al., 2006; Dobbie and Fryer, 2013; Deming et al., 2014)
- Assessments of value-added models (Rothstein, 2010, 2014; Chetty et al., 2014; Kane et al., 2013; Deming, 2014)
- Experimental vs. non-experimental estimators (LaLonde, 1986; Dehija and Wahba, 1999, 2002; Smith and Todd, 2005)
- Empirical Bayes estimation and model uncertainty (Morris, 1983; Judge and Mittlehammer, 2003, 2004, 2007)

Conceptual Framework

- Potential outcomes model:

$$Y_{ij} = \mu_j + a_i$$

- Y_{ij} is potential test score of student i if she attends school j
- μ_j is mean potential outcome at school j
- a_i is student ability
- D_{ij} is a dummy for attendance at school j
- Observed score: $Y_i = \sum_j D_{ij} Y_{ij}$
- Constant effects assumption facilitates our focus on value-added vs. omitted variables bias

Conceptual Framework

- Student ability depends on observables and unobservables:

$$a_i = X_i'\gamma + \epsilon_i$$

- $E[\epsilon_i] = 0$, $E[X_i\epsilon_i] = 0$ by definition
- Observed score can be written

$$Y_i = \mu_0 + \sum_j \beta_j D_{ij} + X_i'\gamma + \epsilon_i$$

- $\beta_j \equiv \mu_j - \mu_0$ is school j's value-added: the causal effect of j relative to omitted reference school 0

Conceptual Framework

$$Y_i = \mu_0 + \sum_j \beta_j D_{ij} + X_i'\gamma + \epsilon_i$$

- Define $b_j \equiv E[\epsilon_i | D_{ij} = 1]$
- b_j is the bias in the OLS estimate for school j
- Selection on observables requires $b_j = 0 \ \forall j$
- More generally, both value-added and bias may vary across schools
- Think of these parameters as (correlated) random effects, with a joint distribution across schools:

$$(\beta_j, b_j) \sim F(\beta, b)$$

Conceptual Framework

$$Y_i = \mu_0 + \sum_j \beta_j D_{ij} + X_i' \gamma + \epsilon_i$$

- With instruments for each school, we can estimate this equation by either OLS or IV:

$$\hat{\beta}_j^{IV} = \beta_j + e_j^{IV}$$

$$\hat{\beta}_j^{OLS} = \beta_j + b_j + e_j^{OLS}$$

- The e_j are measurement errors that vanish as within-school samples tend to infinity
- We use the joint distribution of $\hat{\beta}_j^{IV}$ and $\hat{\beta}_j^{OLS}$ to:
  1. Estimate the joint distribution of β_j and b_j
  2. Generate better estimates of individual β_j

Setting and Data

- We apply our methods to public schools in Boston, MA
- Boston public schools are diverse, with several competing sectors:
  - Traditional district schools
  - Charter schools
  - Pilot schools
  - Exam schools
- Admission processes differ by sector:
  - Traditional and pilot schools: Centralized assignment mechanism
  - Charters: Independent lotteries
  - Exams: Test-based admissions
- In previous work, we've assembled a set of quasi-experiments from each admission process (Abdulkadiroglu et al., 2011, 2014; Angrist et al., 2013a, 2013b)
- Here we unify these studies of individual sectors

Data

- Data comes from four sources:
  - State administrative data on demographics, school attendance, standardized test scores
  - Applications to BPS centralized assignment mechanism
  - Charter lottery records
  - Exam school applications and entrance scores
- Basic sample: Students in Boston at baseline (5th or 8th grade) from 2006-2012
- Two subsamples:
  - OLS sample: All students with followup data
  - IV sample: Students in assignment "strata" with random variation (oversubscribed BPS first choices, charter lotteries, or entrance scores in the neighborhood of an exam cutoff)
- We study schools for which there is at least one quasi-experiment.

  Undersubscribed schools are treated as a composite omitted category

Table 1a: Observational and quasi-experimental school samples, middle school

Sector	School	Students ever enrolled: Observational	Quasi-experimental
Exam	O'Bryant	603	563
	BLA	972	748
	BLS	1,102	577
Charter	APR	313	269
	Boston Col	332	275
	Boston Prep	386	282
	Edward Brooke	215	138
	Excel	224	147
	MATCH	319	230
	Roxbury Prep	447	318
	UP Academy	321	185
Pilot	Frederick	1,129	634
	Harbor	531	389
	Lyndon	277	126
	TechBoston	397	328
Traditional Public	BTU	214	199
	Curley	665	364
	Edison	772	367
	Irving	1,179	704
	Jackson/Mann	474	149
	Lewenberg	293	155
	Mario Umana	792	350
	McCormack	1,341	723
	Mildred	773	431
	Murphy	536	252
	Ohrenberger	413	146
	Perry	185	121
	Quincy	731	216
	Rogers	1,115	665
	Timilty	1,422	1,099
	Warren	291	112
Omitted BPS schools:		22	22
% of students in omitted BPS schools:		19.96%	9.14%

Table 1b: Observational and quasi-experimental school samples, high school

Sector	School	Students ever enrolled: Observational	Students ever enrolled: Quasi-experimental
Exam	O'Bryant	1,627	908
	BLA	1,833	360
	BLS	2,432	141
Charter	BGA	293	135
	CoaH	563	289
	Codman	340	157
	MATCH	457	186
Pilot	ACC	428	234
	BCLA	731	403
	TechBoston	484	288
Traditional Public	Brighton	1,388	882
	Brook Farm	516	317
	English	716	276
	Excel	622	365
	Fenway	574	330
	MCT	500	284
	Madison Park	1,984	392
	New Mission	470	218
	Parkway	466	281
	Snowden	674	427
	Social Justice	439	189
Omitted BPS schools:		22	22
% of students in omitted BPS schools:		23.72%	21.98%

Table 2: Descriptive statistics

		Middle school			High school	
		Boston 5th graders	+ BPS "changer" or 6th grade charter applicant	+ in a strata with instrument variation	Boston 8th graders	+ BPS "changer" or 9th grade charter applicant	+ in a strata with instrument variation
Baseline demographics		(1)	(2)	(3)	(4)	(5)	(6)
Hispanic		0.357	0.367	0.347	0.318	0.386	0.367
Black		0.411	0.419	0.467	0.415	0.429	0.437
White		0.118	0.091	0.081	0.142	0.085	0.085
Asian		0.073	0.085	0.066	0.088	0.063	0.078
Female		0.481	0.503	0.507	0.495	0.499	0.512
Free/reduced price lunch		0.808	0.848	0.838	0.741	0.829	0.816
Special education		0.243	0.191	0.186	0.205	0.212	0.190
Limited English proficient		0.229	0.244	0.208	0.137	0.179	0.145
	N	31,569	15,893	10,289	40,576	21,112	12,661
Baseline test scores							
Math		-0.475	-0.411	-0.417	-0.337	-0.569	-0.455
	N	29,992	15,737	10,206	38,359	20,607	12,459
ELA		-0.593	-0.548	-0.530	-0.441	-0.660	-0.540
	N	29,582	15,590	10,159	37,911	20,355	12,371

Observational Model

- Estimating equation for observational (OLS) analysis:

$$Y_i = \alpha + \sum_j \beta_j D_{ij} + X_i'\gamma + \epsilon_i \tag{1}$$

- Y_i is a 7th- or 10th-grade test score for student i
- The D_{ij} measure years of exposure to each school
- X_i is a vector of standard VAM covariates: gender, race, subsidized lunch, limited English proficiency, special education, baseline math and English language arts (ELA) scores

Quasi-experimental Model

- Two-stage least squares (2SLS) system for quasi-experimental analysis:

$$Y_i = \sum_j \beta_j D_{ij} + \sum_\ell Q_{i\ell} \left(\alpha_\ell + C_{i\ell}'\theta_\ell\right) + X_i'\gamma + \epsilon_i \qquad (2)$$

$$D_{ik} = \sum_j \pi_{jk} Z_{ij} + \sum_\ell Q_{i\ell} \left(\tau_{\ell k} + C_{i\ell}'\kappa_{\ell k}\right) + X_i'\delta_k + \eta_{ik} \qquad (3)$$

- $Q_{i\ell}$ is a dummy equal to one if student i is in quasi-experimental sample ℓ
- $C_{i\ell}$ is a vector of *design controls* for experiment ℓ: Dummies for lottery randomization strata, or polynomial in exam school running variable
- Z_{ij} is an offer ("qualification") instrument for school j. This dummy is equal to zero for all students not in a quasi-experimental sample for school j

Figure 1: School-specific first stages

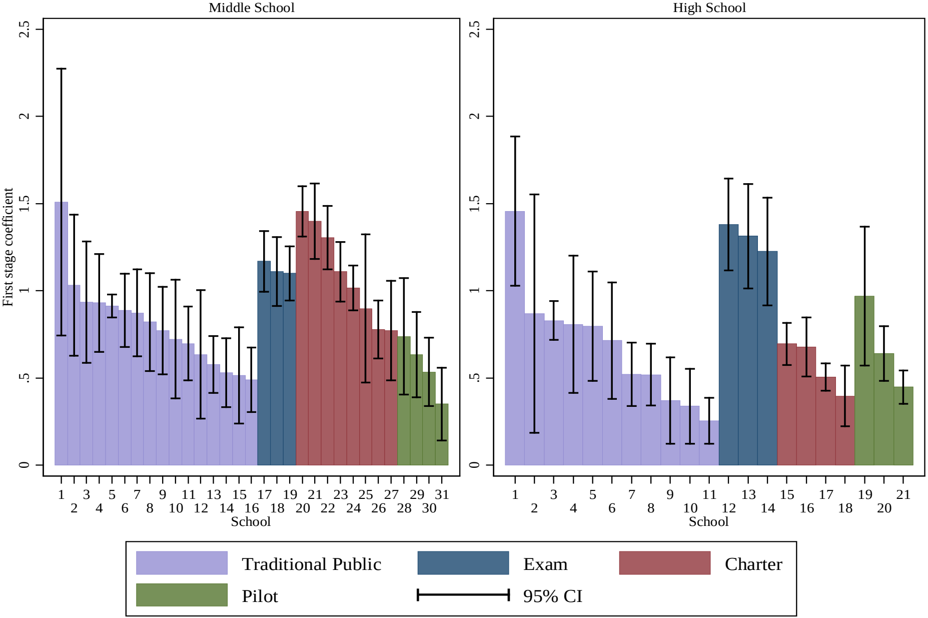


Figure 2a: Coefficients on school 3 qualification in all school's first stages (including composite)

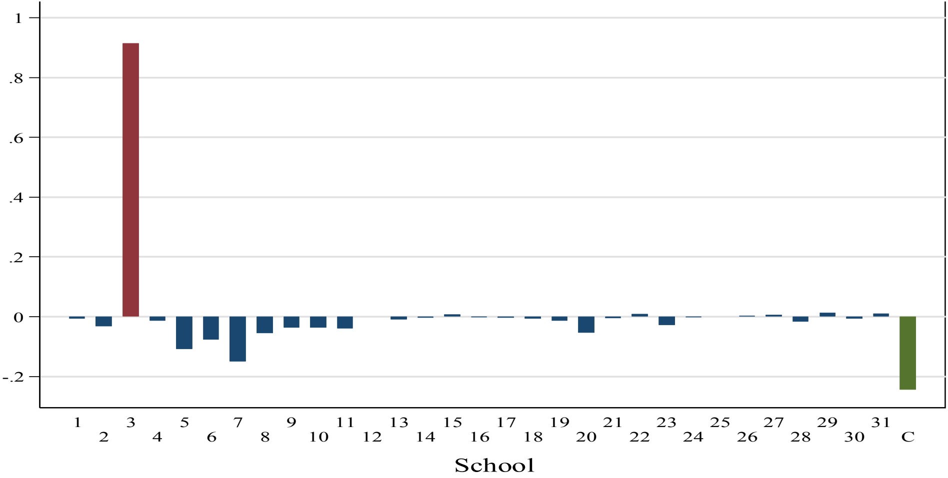


Figure 2b: Coefficients on school 3 qualification and other qualifications in the first stage of School 3

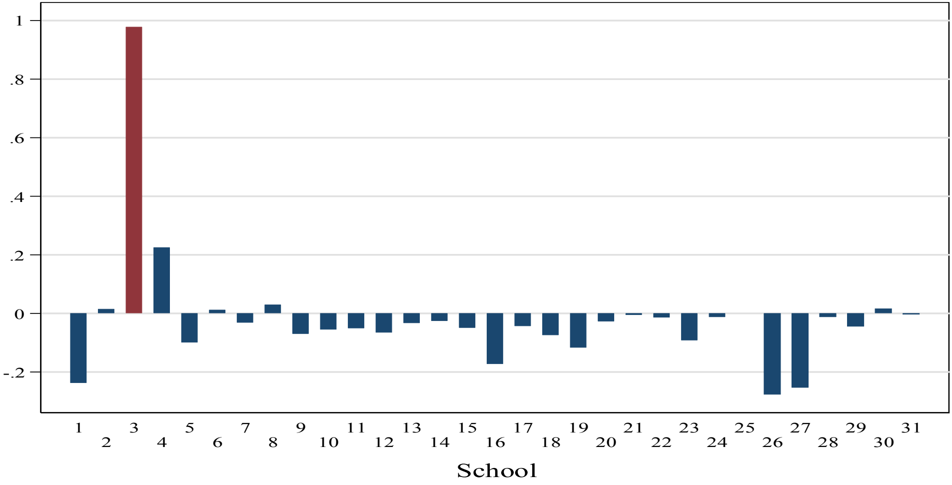
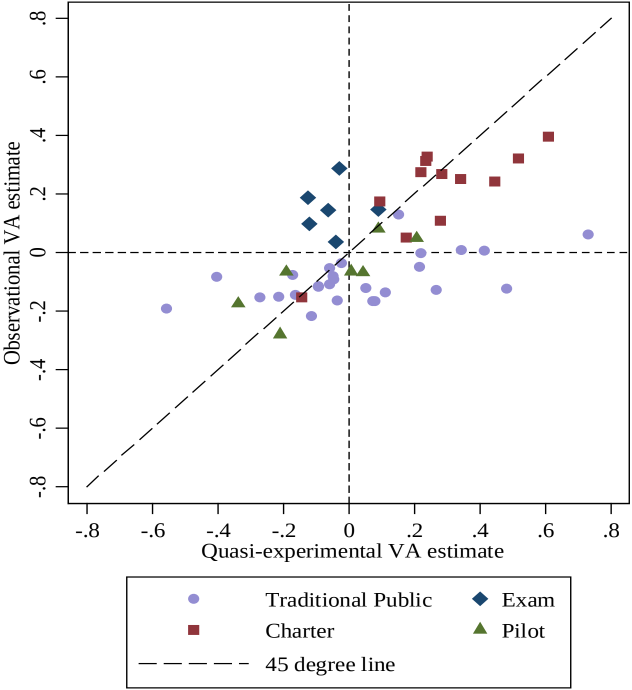


Figure 3a: Observational and quasi-experimental math value-added estimates, by sector



Bias and Value-added Distributions

- OLS and 2SLS yield two estimates for each school:

$$\hat{\beta}_j^{IV} = \beta_j + e_j^{IV}$$

$$\hat{\beta}_j^{OLS} = \beta_j + b_j + e_j^{OLS}$$

- Next, model value-added and bias as a function of school characteristics W_j, including sector effects:

$$E[\beta_j | W_j] = W_j' \psi_\beta, \quad E[b_j | W_j] = W_j' \psi_b$$

- With $B_j = (\beta_j, \beta_j + b_j)'$ and $\psi = (\psi_\beta, \psi_\beta + \psi_b)'$, write

$$E\left[(B_j - \psi W_j)(B_j - \psi W_j)' | W_j\right] = \begin{bmatrix} \sigma_\beta^2 & \sigma_\beta^2 + \sigma_{\beta b} \\ \sigma_\beta^2 + \sigma_{\beta b} & \sigma_\beta^2 + 2\sigma_{\beta b} + \sigma_b^2 \end{bmatrix}$$
$$\equiv \Gamma$$

- ψ and Γ are *hyperparameters* governing distributions of value-added and bias

FGLS Estimation

- Write the observed estimates $\hat{B}_j = \left(\hat{\beta}_j^{IV}, \hat{\beta}_j^{OLS}\right)'$ as

$$\hat{B}_j = \psi W_j + u_j \tag{4}$$

- The residuals satisfy $E[u_j|W_j] = 0$, and

$$E[u_j u_j'|W_j] = \Gamma + \Lambda_j$$

- Λ_j is the covariance matrix of IV and OLS sampling errors, e_j^{IV} and e_j^{OLS}
- We estimate Λ_j using standard asymptotic theory for IV and OLS
- Then estimate equation (4) by FGLS
- Use residuals to estimate Γ, and back out σ_β^2, σ_b^2 and $\sigma_{\beta b}$
- This approach requires IV asymptotics to accurately approximate the distribution of e_j^{IV}

Table 4a: Math hyperparameter estimates

	Unweighted	One-step FGLS	Iterated FGLS
VA shifters	(1)	(2)	(3)
Traditional public	0.031	-0.023	-0.021
	(0.052)	(0.046)	(0.047)
Exam	-0.045	-0.061	-0.060
	(0.082)	(0.079)	(0.081)
Charter	0.277***	0.232***	0.235***
	(0.060)	(0.055)	(0.056)
Pilot	-0.054	-0.085	-0.083
	(0.086)	(0.082)	(0.083)
High school	-0.050	-0.024	-0.026
	(0.147)	(0.137)	(0.139)
Bias shifters			
Traditional public	-0.104**	-0.050	-0.052
	(0.043)	(0.036)	(0.037)
Exam	0.218***	0.232***	0.232***
	(0.060)	(0.057)	(0.059)
Charter	-0.047	-0.001	-0.004
	(0.045)	(0.039)	(0.041)
Pilot	-0.002	0.028	0.027
	(0.069)	(0.064)	(0.066)
High school	0.178	0.153	0.154
	(0.115)	(0.101)	(0.105)
Variance components			
VA std. dev.	0.156***	0.161***	0.160***
	(0.036)	(0.036)	(0.036)
Bias std. dev	0.090*	0.097**	0.097*
	(0.052)	(0.050)	(0.050)
VA, bias correlation	-0.812***	-0.818***	-0.818***
	(0.097)	(0.103)	(0.102)
N (schools)		52	

Figure 4a: FGLS math value-added estimates, by sector

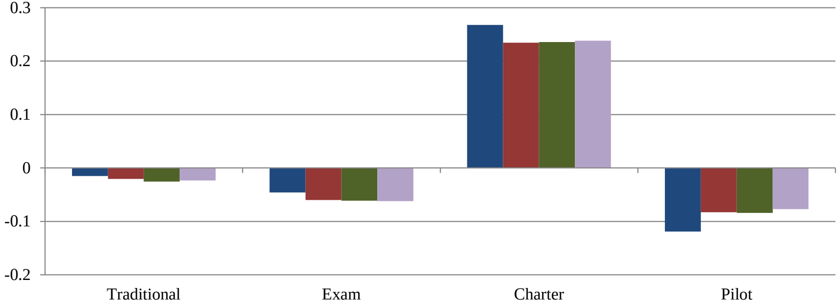
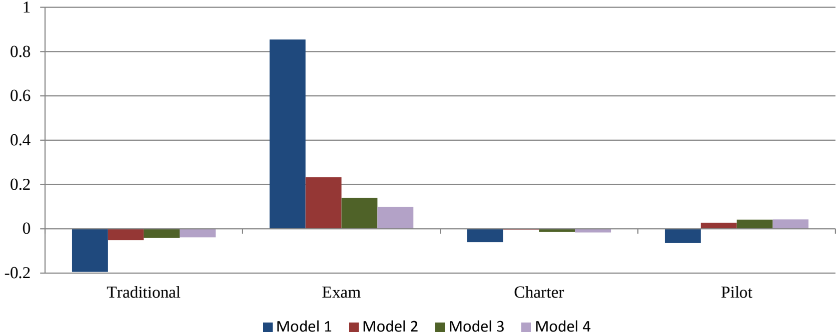


Figure 4b: FGLS math bias estimates, by sector



Minimum MSE Predictions

- To produce estimates for individual schools, add parametric structure:

$$(\beta_j, b_j)|W_j \sim N\left((W_j'\psi_\beta, W_j'\psi_b), \Delta\right)$$

$$\left(e_j^{IV}, e_j^{OLS}\right)|\beta_j, b_j, W_j \sim N\left(0, \Lambda_j\right)$$

- Then posterior distribution for parameters at school j is

$$(\beta_j, b_j)|\hat{\beta}_j^{IV}, \hat{\beta}_j^{OLS}, W_j \sim N\left(\left(\beta_j^*, b_j^*\right), V_j^*\right)$$

- Posterior mean for β_j is

$$\beta_j^* = w_{1j}\hat{\beta}_j^{IV} + w_{2j}\left(\hat{\beta}_j^{OLS} - W_j'\psi_b\right) + (1 - w_{1j} - w_{2j})W_j'\psi_\beta$$

- Weights w_{1j} and w_{2j} depend on Λ_j and Δ
- β_j^* is MSE-minimizing function of $\hat{\beta}_j^{IV}$ and $\hat{\beta}_j^{OLS}$
- Empirical Bayes (EB) posterior mean plugs in estimates of ψ_β, ψ_b, and Δ

Minimum MSE Weights

$$\beta_j^* = w_{1j}\hat{\beta}_j^{IV} + w_{2j}\left(\hat{\beta}_j^{OLS} - W_j'\psi_b\right) + (1 - w_{1j} - w_{2j})W_j'\psi_\beta$$

- Posterior mean is a weighted average of three things:
  1. The unbiased IV estimate
  2. The biased OLS estimate, net of mean bias
  3. The prior mean
- Shrinkage toward the mean comes from standard Bayesian logic
- Weights sum to one, but are not always between 0 and 1
- OLS weight can exceed 1 when $Cov(\beta_j, b_j) < 0$ and $\sigma_b < \sigma_\beta$
- Empirically, lots of variation in weight assigned to IV vs. OLS

Figure 5a: Minimum MSE weights on observational and quasi-experimental math VA estimates, by sector

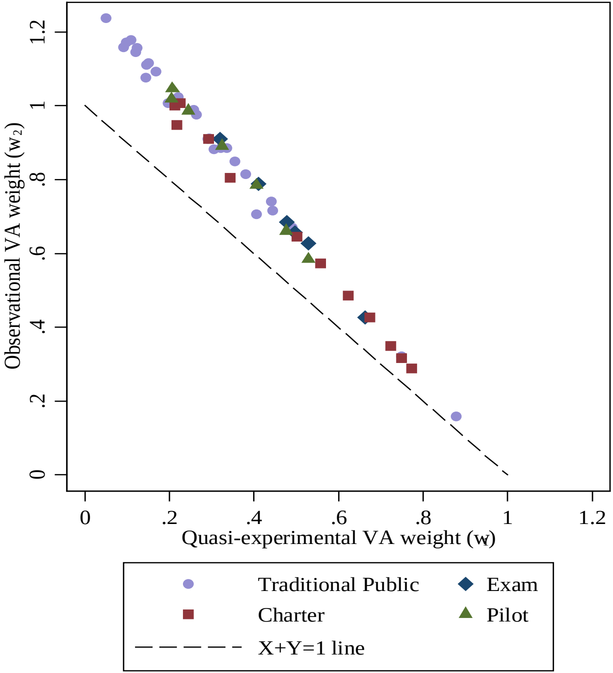


Figure 6a: Minimum MSE, observational, and quasi-experimental math VA estimates, by sector

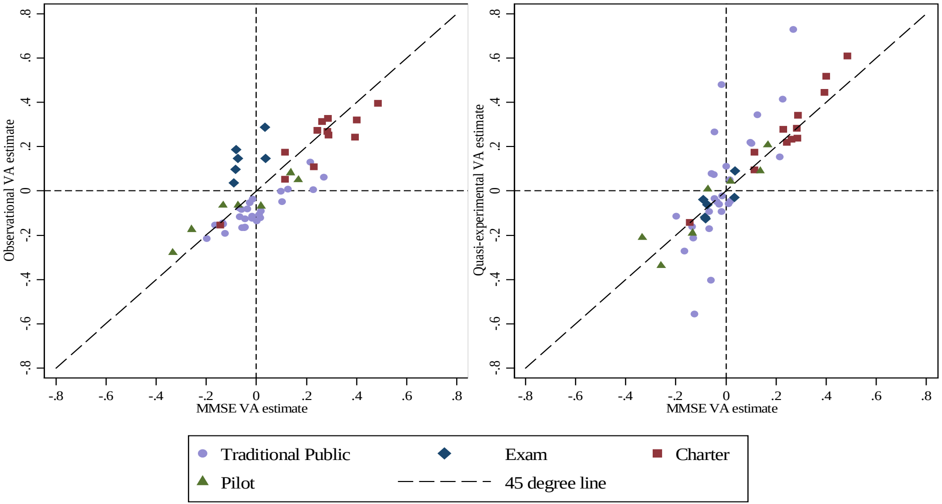


Table 5: Root Mean Squared Error of Value-added Estimators

		Shrunk	
	Unshrunk	No sector effects	With sector effects
	(1)	(2)	(3)
OLS	0.167	0.167	0.168
IV	0.161	0.115	0.112
MMSE	-	0.099	0.085

Notes: This table reports root mean squared error (RMSE) for school value-added estimators. Models in column (2) shrink school-specific estimates towards the overall mean value-added. Models in column (3) shrink the estimates towards sector mean value-added.

Improvements in Policy

- How much do these improvements in MSE matter?
- We simulate data from a model calibrated to match our Boston estimates
- Then rank schools according to estimated value-added using each method
- Compare misclassification rates for two policies:
  1. Close failing schools (bottom quintile)
  2. Expand successful schools (top quintile)

Table 6: Accuracy of Policies Based on Value-added Models

	Close failing schools		Expand successful schools	
	Fraction of failing schools not classified as failing	Fraction of non-failing schools classified as failing	Fraction of successful schools not classified as successful	Fraction of unsuccessful schools classified as successful
Estimator	(1)	(2)	(3)	(4)
OLS	0.494	0.124	0.417	0.104
Shrunk OLS	0.499	0.125	0.419	0.105
IV	0.370	0.093	0.325	0.081
Shrunk IV	0.374	0.094	0.255	0.064
MMSE	0.270	0.067	0.206	0.051

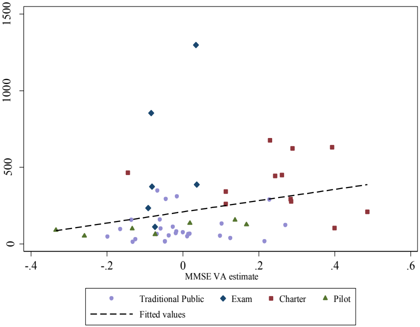
Notes: This table describes the effects of policies that close or expand schools based on measures of school value-added. Columns (1) and (2) assess a policy designed to close failing schools, defined as schools in the bottom quintile of value-added. Columns (3) and (4) assess a policy designed to expand successful schools, defined as those in the top quintile. The results come from 10,000 simulations of a model calibrated to match estimates from Boston data. Shrunk and MMSE models compute posterior means by shrinking school-specific estimates towards sector means.

Table A4: Correspondence Between Correct and Assigned Report Card Grades

		Correct grade				
		A	B	C	D	F
Assigned grade	Estimator	(1)	(2)	(3)	(4)	(5)
A	OLS	0.583	0.272	0.110	0.032	0.003
	Shrunk OLS	0.581	0.271	0.111	0.034	0.003
	IV	0.675	0.246	0.058	0.015	0.006
	Shrunk IV	0.745	0.179	0.056	0.017	0.003
	MMSE	0.794	0.180	0.024	0.002	0.000
B	OLS	0.197	0.311	0.268	0.179	0.044
	Shrunk OLS	0.198	0.311	0.262	0.184	0.046
	IV	0.198	0.412	0.269	0.085	0.036
	Shrunk IV	0.206	0.427	0.242	0.099	0.026
	MMSE	0.182	0.517	0.248	0.048	0.005
C	OLS	0.102	0.191	0.280	0.263	0.164
	Shrunk OLS	0.103	0.191	0.278	0.262	0.166
	IV	0.076	0.207	0.385	0.231	0.102
	Shrunk IV	0.037	0.254	0.370	0.243	0.097
	MMSE	0.020	0.231	0.465	0.239	0.046
D	OLS	0.069	0.114	0.246	0.254	0.316
	Shrunk OLS	0.070	0.114	0.249	0.250	0.317
	IV	0.032	0.085	0.254	0.375	0.254
	Shrunk IV	0.007	0.090	0.273	0.352	0.277
	MMSE	0.002	0.045	0.264	0.448	0.242
F	OLS	0.035	0.084	0.152	0.223	0.506
	Shrunk OLS	0.036	0.085	0.157	0.222	0.501
	IV	0.010	0.026	0.087	0.247	0.630
	Shrunk IV	0.001	0.023	0.110	0.240	0.626
	MMSE	0.000	0.004	0.048	0.218	0.730

Value-added, Bias, and Oversubscription

- Do parents value school quality, or bias? (Rothstein 2006)
- We compute school oversubscription rates (number of first-choice applications for traditional publics, pilots and exams; number of total applications for charters)
- Then examine relationship between oversubscription and EB posterior estimates
- Results: Oversubscription positively correlated with both value-added and bias



1500
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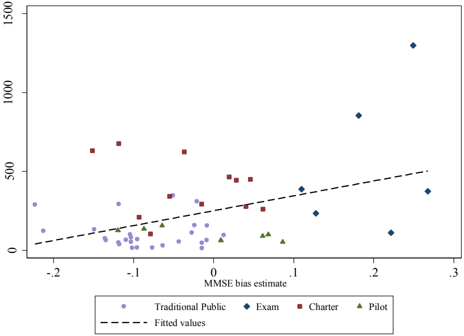
.6
MMSE VA estimate
Traditional Public
Exam
Charter
Pilot
Fitted values



1500
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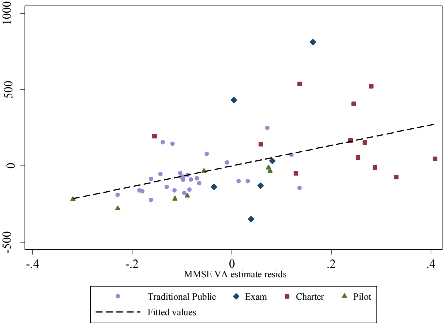
MMSE bias estimate
Traditional Public
Exam
Charter
Pilot
Fitted values



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MMSE VA estimate resids
Traditional Public
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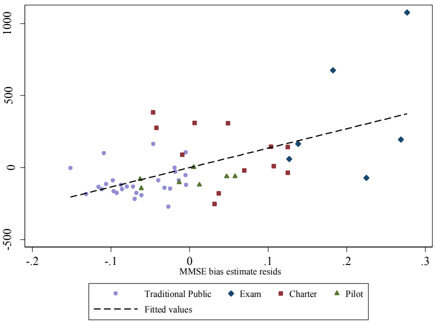
Charter
Pilot
Fitted values



1000
500
0
-500
-.2
-.1
0
.1
.2
.3
MMSE bias estimate resids
Traditional Public
Exam
Charter
Pilot
Fitted values

Conclusion

- This project uses school admissions lotteries to validate and improve upon observational school value-added models
- Estimates from Boston show bias in observational value-added both within and between school sectors
- Our findings establish the value of lottery-based VAMs for research and policy
- Hybrid strategies improve policy targeting relative to either observational or lottery estimates alone

Figure 3b: Observational and quasi-experimental ELA value-added estimates, by sector

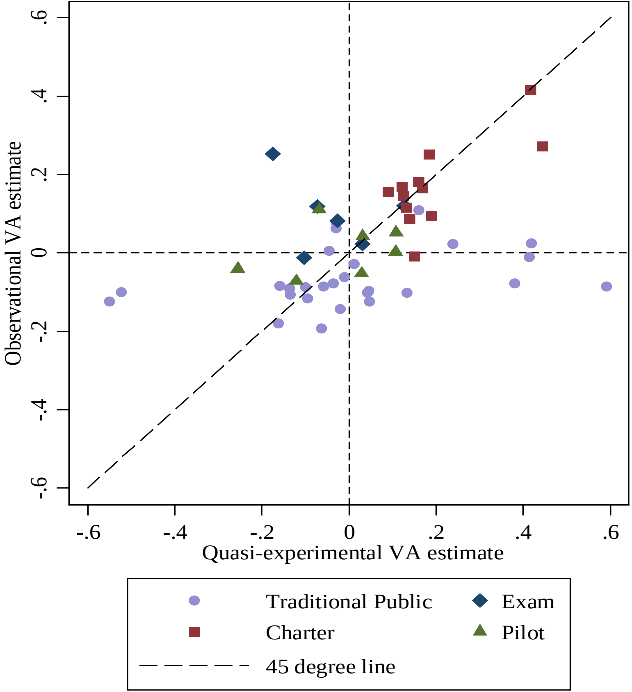


Table 4b: ELA hyperparameter estimates

	Unweighted	One-step FGLS	Iterated FGLS
VA shifters	(1)	(2)	(3)
Traditional public	0.028	-0.036	-0.033
	(0.045)	(0.035)	(0.037)
Exam	-0.017	-0.046	-0.047
	(0.060)	(0.056)	(0.059)
Charter	0.207***	0.162***	0.164***
	(0.048)	(0.038)	(0.041)
Pilot	-0.007	-0.047	-0.046
	(0.069)	(0.062)	(0.065)
High school	-0.056	-0.010	-0.012
	(0.117)	(0.099)	(0.105)
Bias shifters			
Traditional public	-0.080*	-0.015	-0.019
	(0.043)	(0.032)	(0.034)
Exam	0.132**	0.161***	0.162***
	(0.054)	(0.050)	(0.053)
Charter	-0.025	0.021	0.020
	(0.044)	(0.033)	(0.037)
Pilot	0.028	0.066	0.065
	(0.065)	(0.057)	(0.060)
High school	0.136	0.091	0.093
	(0.109)	(0.090)	(0.096)
Variance components			
VA std. dev.	0.086	0.097*	0.096*
	(0.055)	(0.051)	(0.051)
Bias std. dev	0.062	0.077	0.076
	(0.073)	(0.061)	(0.061)
VA, bias correlation	-0.496	-0.630	-0.623
	(1.139)	(0.875)	(0.887)
N (schools)		52	

Figure 5b: Minimum MSE weights on observational and quasi-experimental ELA VA estimates, by sector

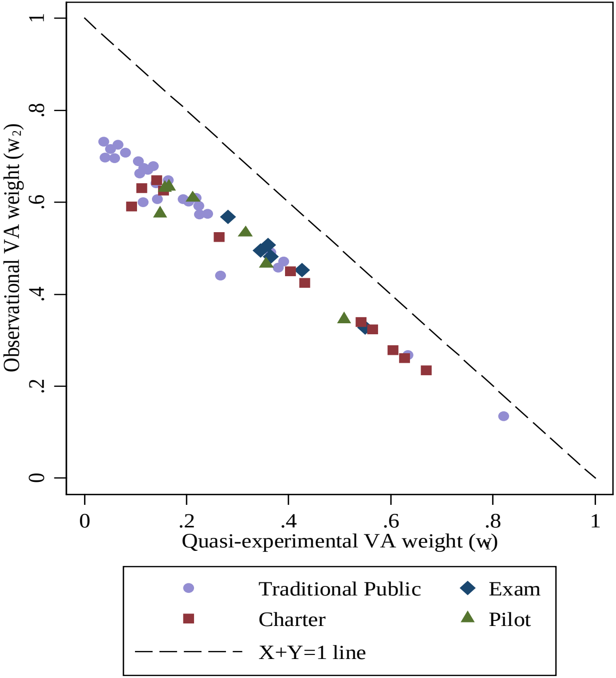


Table A1: Covariate balance for qualification instruments

		Qualification instrument balance (5th, 6th, and 7th grade entry samples)					Qualification instrument balance (9th grade entry sample)				
		Any qualification	Exam	Charter	Pilot	Traditional public	Any qualification	Exam	Charter	Pilot	Traditional public
Baseline demographics		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Hispanic		0.016	-0.040	0.039**	-0.031	0.017	-0.010	-0.073	-0.006	0.026	-0.011
		(0.012)	(0.049)	(0.016)	(0.025)	(0.014)	(0.014)	(0.056)	(0.022)	(0.021)	(0.014)
Black		-0.019	0.005	-0.037**	0.015	-0.012	0.004	-0.004	0.005	-0.019	0.015
		(0.012)	(0.051)	(0.017)	(0.028)	(0.014)	(0.014)	(0.056)	(0.023)	(0.021)	(0.014)
White		-0.002	0.024	-0.006	0.015	-0.005	0.006	0.031	0.009	0.009	0.002
		(0.007)	(0.048)	(0.012)	(0.012)	(0.007)	(0.008)	(0.041)	(0.011)	(0.012)	(0.008)
Asian		0.008	0.031	0.007	0.012	0.007	-0.003	0.025	-0.006	-0.025**	-0.006
		(0.006)	(0.053)	(0.007)	(0.012)	(0.006)	(0.007)	(0.054)	(0.008)	(0.011)	(0.007)
Female		0.014	-0.004	0.014	0.016	0.011	-0.002	0.022	-0.027	0.019	0.008
		(0.013)	(0.059)	(0.018)	(0.029)	(0.015)	(0.014)	(0.062)	(0.023)	(0.022)	(0.015)
Free/reduced price lunch		0.014	-0.031	0.005	-0.012	0.018*	0.005	0.034	0.022	-0.009	-0.006
		(0.010)	(0.054)	(0.016)	(0.020)	(0.010)	(0.011)	(0.051)	(0.019)	(0.017)	(0.011)
Special education		0.008	0.023	0.017	-0.030	0.004	0.009	0.007	-0.024	0.023	0.007
		(0.010)	(0.020)	(0.014)	(0.023)	(0.011)	(0.010)	(0.022)	(0.019)	(0.016)	(0.011)
Limited English proficient		-0.006	0.007	0.002	0.008	-0.003	0.001	0.044	-0.012	-0.018	0.008
		(0.009)	(0.024)	(0.014)	(0.020)	(0.011)	(0.008)	(0.031)	(0.014)	(0.012)	(0.009)
	N	14,121	1,216	4,692	1,978	8,357	12,448	1,029	2,626	3,484	9,051
Baseline test scores											
Math		0.001	-0.038	-0.023	0.053	0.000	-0.005	0.062	0.044	-0.008	-0.011
		(0.023)	(0.052)	(0.035)	(0.053)	(0.025)	(0.024)	(0.064)	(0.041)	(0.039)	(0.026)
	N	13,962	1,209	4,611	1,959	8,291	12,263	1,019	2,598	3,445	8,902
ELA		-0.008	-0.076	-0.018	0.037	0.011	-0.016	-0.017	0.049	-0.012	-0.032
		(0.024)	(0.063)	(0.037)	(0.054)	(0.027)	(0.025)	(0.067)	(0.042)	(0.041)	(0.027)
	N	13,907	1,211	4,592	1,951	8,252	12,178	1,015	2,593	3,427	8,841

Table A2: Attrition, middle school

		Sample means (6th grade entry sample)			Qualification instrument balance (5th, 6th, and 7th grade entry samples)				
		Boston 5th graders	+ BPS "changer" or 6th grade charter applicant	+ in a strata with instrument variation	Any qualification	Exam	Charter	Pilot	Traditional public
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Has 7th grade state math score		0.863	0.907	0.902	0.009	-0.027	-0.001	-0.007	0.017*
					(0.009)	(0.028)	(0.016)	(0.020)	(0.010)
Has 7th grade state ELA score		0.865	0.908	0.903	0.013	-0.024	0.002	-0.012	0.021**
					(0.009)	(0.028)	(0.016)	(0.020)	(0.010)
	N	23,892	12,569	8,326	10,604	1,216	2,691	1,634	6,768
In Boston up to 7th grade		0.918	0.936	0.936	0.013*	0.029	0.013	0.004	0.017**
					(0.007)	(0.018)	(0.010)	(0.014)	(0.008)
	N	25,261	13,304	8,758	11,273	1,177	3,203	1,741	7,060
Has 8th grade state math score		0.838	0.882	0.879	0.023**	0.008	0.004	-0.002	0.032***
					(0.011)	(0.035)	(0.021)	(0.023)	(0.011)
Has 8th grade state ELA score		0.839	0.882	0.879	0.023**	0.013	0.008	-0.009	0.032***
					(0.011)	(0.034)	(0.020)	(0.023)	(0.011)
	N	19,781	10,755	7,150	9,119	1,216	1,757	1,438	5,962
In Boston up to 8th grade		0.890	0.911	0.911	0.017*	0.049**	0.017	0.013	0.022**
					(0.009)	(0.023)	(0.013)	(0.019)	(0.010)
	N	20,844	11,294	7,423	9,385	1,140	2,230	1,465	6,057

Table A3: Attrition, high school

	Sample means (9th grade entry sample)			Qualification instrument balance (9th grade entry sample)				
	Boston 8th graders	+ BPS "changer" or 9th grade charter applicant	+ in a strata with instrument variation	Any qualification	Exam	Charter	Pilot	Traditional public
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Has 10th grade state math score	0.758	0.771	0.784	-0.005	-0.016	0.000	0.001	-0.007
				(0.013)	(0.040)	(0.020)	(0.019)	(0.014)
Has 10th grade state ELA score	0.768	0.785	0.795	0.002	-0.023	0.002	-0.006	-0.002
				(0.013)	(0.040)	(0.020)	(0.019)	(0.014)
N	31,328	16,021	10,450	10,264	1,029	2,074	2,729	7,463
In Boston up to 10th grade	0.917	0.927	0.922	0.006	0.031	0.042***	-0.004	-0.001
				(0.009)	(0.030)	(0.015)	(0.015)	(0.009)
N	29,822	15,666	9,999	9,829	922	2,225	2,659	7,041